**Appendix A. CNP fluxes**

The fluxes of carbon, nitrogen, and phosphorus coming from the upstream pool (*i*) to the downstream pool (*j*) due to SOM decomposition are calculated as:

 (A1)

 (A2)

 (A3)

where *gi* is the percentage of carbon remaining in the soil after decomposition of the *ith* SOM pool (*i.e.*, CUE, with the rest being released as CO2); *fij* is the fraction of SOM leaving the *ith* pool and entering the *jth* pool; and  is the first order decay of the *ith* SOM pool. CN and CP are soil C:N and C:P ratios.

If the upstream-decomposed soil organic nitrogen (phosphorus) is more than enough to sustain the downstream C:N (C:P) ratio, then the excess nitrogen (phosphorus) enters the soil NH4+ (POx) pool. POx represents the sum of PO43-, HPO42-, and H2PO4- that could be utilized by plants and microorganisms, and adsorbed by mineral surface.

 (A4)

 (A5)

where *FmobN,ij* and *FmobP,ij* are the nitrogen and phosphorus gross mineralization rates. Eqn. A4 - A5 ensure that gross mineralization is not less than zero. In contrast, if nitrogen (phosphorus) is insufficient, soil microbes immobilize free NH4+ and NO3- (POx) according to uptake kinetics:

 (A6)

 (A7)

 (A8)

where , , and  are microbial NH4+, NO3-, and POx uptake rates, respectively. *VMAX* and *ECA* are maximum uptake rates and *ECA* competition terms, respectively. The soil CNP stoichiometry is flexible and depends on the predicted immobilization rates. In addition to microbial uptake, plants also consume a portion of soil nutrients, which is modeled analogously to the approach described above for microbial immobilization:

 (A9)

 (A10)

 (A11)

where , , and  are plant NH4+, NO3-, and POx uptake rates, respectively. Other soil nutrient consumers are nitrifiers, denitrifiers, and mineral surfaces:

 (A12)

 (A13)

 (A14)

where , , and  are NH4+ nitrification, NO3- denitrification, and mineral surface POx adsorption rates, respectively; [NH4] is the free NH4+ pool; and  is the maximum fraction of free NH4+ pool that could be utilized by nitrifiers. The nitrification rate is controlled by soil temperature () and soil moisture (), soil oxygen status (*1-fanox*), and a competition factor (). The denitrificaiton rate () is either constrained by substrate availability (*f(decomp)*) or NO3- availability (*f([NO3])*) [*Del Grosso et al.*, 2000], taking into account the soil anaerobic condition (*fanox*) and competition (*ECAdenNO3*). **is derived from the Langmuir adsorption model [*Barrow*, 1978], where actual adsorption P is equal to . Taking the time derivative leads to a relationship for the adsorption rate [*Wang et al.*, 2010].

Soil NH4+ content is altered by inputs from deposition () and biological N2 fixation (*FBNF*) [*Cleveland et al.*, 1999]:

 (A15) where *NPPannual* is annual net primary production. Controls on biological N2 fixation are complex and several models have been developed for large-scale land BGC models [*Cleveland et al.*, 1999; *Fisher et al.*, 2010; *Hartwig*, 1998; *Parton et al.*, 1993; *Running et al.*, 1989; *Vitousek and Field*, 1999]. However, the emergent responses predicted across these model structures are inconsistent [*Galloway et al.*, 2004]. Recognizing this important structural uncertainty, we used a simple model where biological N2 fixation (*FBNF*) is modeled as a function of annual NPP [*Cleveland et al.*, 1999].

Soil NO3- content is modified by external deposition inputs () and leaching losses ():

 (A16)

where soil nitrate concentration ([NO3]: gN m-2) divided by soil water content (W: gH2O m-2) results in the concentration of dissolved nitrate (DIN). The hydrologic discharge (*Qdis*: gH2O m-2 s-1) applied to DIN (gN gH2O-1) leads to the leaching loss (gN m-2 s-1).

Soil POx content is affected by external inputs from parent material weathering (*Fweather*) and leaching losses (). Sorbed P (*PS*) could be further strongly occluded and become unavailable for plant and microbial uptake. Parent material stock can be increased by atmospheric dust deposition () [*Mahowald et al.*, 2008]:

 (A17)

 (A18)

 (A19)

where parent material weathering (*Fweather*) is calculated using a weather rate (*kweather*) and parent material P content ([*PP*]). POx leaching loss is modeled with a similar approach to nitrate leaching (Eqn. A16). Phosphorus occlusion rate is modeled as the product of a constant rate (*koccl*) and the sorbed P content ([*PS*]).

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**Appendix B. Derivation of *VMAX***

The enzyme substrate reaction is:, where the enzyme (*E*) and substrate (*S*) reaction is reversible and forms complex (*C*). The irreversible reaction releases product (*P*) and liberates enzyme (*E*). At steady state, the formation rate of the enzyme substrate complex is equal to the consumption rate:

 (B1)

To simply the equation, we define an affinity parameter:

 (B2)

By definition, the total enzymes  in the system is the sum of free enzymes  and enzymes that are bound with the substrate :

 (B3)

Substituting Eqn. (9) into (8), we have:

 (B4)

Collecting terms containing [C], we have:

 (B5)

The production rate is:

 (B6)

Substituting Eqn. (11) into (12), we have:

 (B7)

Comparing Eqn. (13) with the classic Michaelis-Menten equation, it is clear that the definition of maximum production rate is the product of the reaction rate and enzyme abundance in the system:

 (B8)